Pricing the Pre-termination Option in Deposits: An Adaptation of the Binomial Option Pricing Model

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Bank deposits contain an option which allows the depositor to break the deposit prior to maturity and reinvest the funds at a higher yield. How much is this option worth? This is the question that this paper tries to answer by applying the Binomial Option Pricing Model (BOPM). A price for the pre-termination option was determined and shown to be "correct" in the sense that an arbitrage opportunity is not present at that price. The surprising, counter-intuitive result is that the option price is invariant relative to pre-termination penalties. Why this is so is a matter that may merit further analysis.

1. Introduction

Bank deposits contain an option which allows the depositor to break the deposit prior to maturity. The investor may do this if the funds are needed for some purpose or if he finds an opportunity to reinvest at a higher yield. Savings deposits may be pre-terminated without penalty. In the Philippines, pre-termination of time deposits penalizes the depositor with a lower rate for the period that the funds were actually deposited. If one-half or less of the original period has elapsed, only one-fourth of the original rate will be given. If over one-half has elapsed, one-half of the original rate is paid. In spite of the penalty the option to pre-terminate is still valuable. How much is the option worth? This is the question that this paper tries to answer by applying the Binomial Option Pricing Model (*BOPM*). *BOPM* is often illustrated in the context of stock options. There also have been studies on the use of *BOPM* on options on debt instruments but the author is not aware of any that specifically deals with the pre-termination option.

This paper will show the adaptation of *BOPM* to price the pre-termination option. The paper assumes the investor pre-terminates a deposit to allow him to reinvest at a higher yield. The pre-termination is therefore an exercise of an option on a higher return if that return materializes, i.e., a call option on a higher pay-off. For want of a better term, it will be referred to as a pre-termination option.[1]

What are the practical implications of pricing this option? If an equilibrium price can be established, it is not difficult to see that the option can be traded separately from the deposit. Imagine a bank that issues time deposits. Since the option is inherent in the deposit certificates it issues, it faces the risk that the deposits will be pre-terminated and reinvested at a higher rate. For the bank, this could mean a higher cost of funds. To avoid this risk, the bank could "buy back" the option from the depositor. Effectively this means issuing

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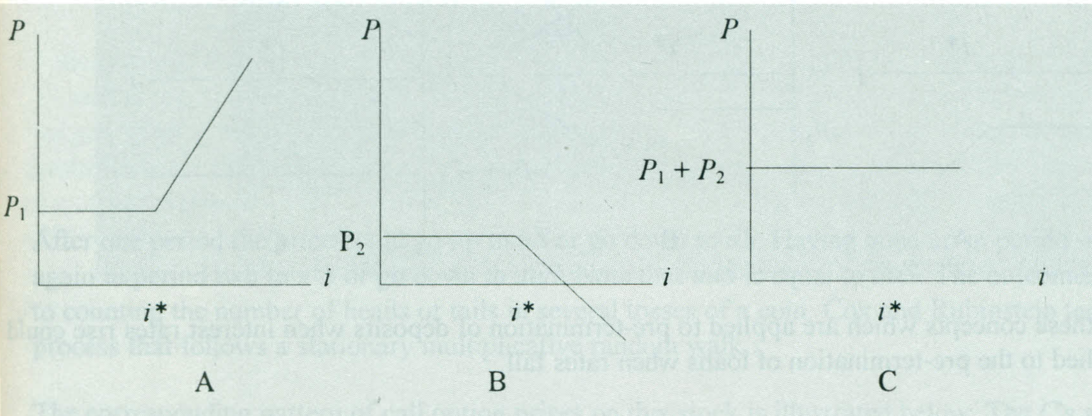
[1] Ybañez and Bautista suggest that the pre-termination option is a put option because it allows the depositor to "force" the bank to buy-back (encash) the deposit. This is correct, but, in the context of pre-terminating to avail of a higher yield, the act of pre-terminating *per se* does not produce the higher pay-off. In the general case of puts, the exercise of the put produces the pay-off. It is the choice of reinvesting that produces the pay-off. This is a fine point that may not be too important because of the symmetrical treatment of calls and puts in *BOPM*, at least for the simple situation analyzed. The *BOPM* formula presented here would work for puts and only a change in perspective or sign is necessary, i.e., instead of the depositor's perspective, take the bank's; instead of selling a call , buy a put.

time deposits without a pre-termination option. Obviously, the depositor will have to be compensated for parting with the option. The depositor "sells" the option to the bank for a price. Selling the option means selling a right to avail of a higher pay-off, i.e., selling a call option.

The situation could be illustrated graphically. These graphs are drawn from the depositor's point of view.

Figure 1 - Depositor's pay-off profile if pre-termination option is sold.



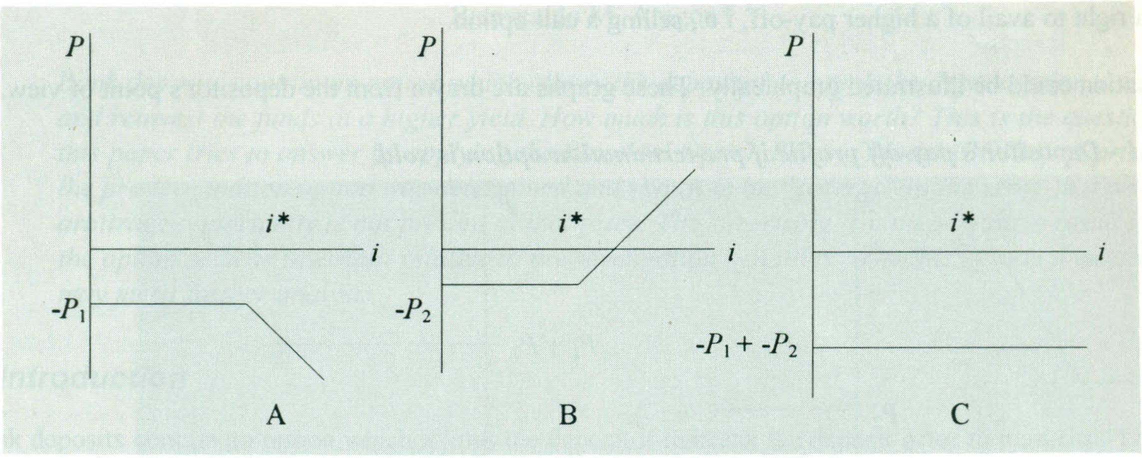
Panel A shows that if interest rates rise beyond $i*$, the option to pre-terminate will result in higher pay-off. $i*$ is the interest rate when the deposit was originally made and the graph is simplified by assuming there is no pre-termination penalty. This assumption is dropped later.

Panel B shows the pay-off from selling the pre-termination option. It is identical to the pay-off profile from selling a call option. Effectively, the investor has sold a call option on the excess earnings arising from the higher yield.

Panel C is the result of adding graphically panels A and B. It shows the pay-off profile that results from a time deposit (with a pre-termination option) combined with a sale of the option. The pay-off is now constant whatever the interest rate may be. Note that the pay-off ($P_1 + P_2$) is higher than what would have been earned from the deposit (P_1) at the original interest rate. The difference is accounted for by the price received from selling the option (P_2). The pay-off profile has been altered from one with a low guaranteed yield with further upside potential to one with a higher guaranteed yield with no upside potential.

From the bank's point of view, the graphs would be the mirror images since the interest represent costs.

Figure 2 - Bank's pay-off profile if pre-termination option is bought.



It is clear that these concepts which are applied to pre-termination of deposits when interest rates rise could be equally applied to the pre-termination of loans when rates fall.

2. The Binomial Option Pricing Model (BOPM)

The Binomial Option Pricing Model was developed by Sharpe, Cox, Rubinstein and Ross. (Cox and Rubinstein (1985), Dubofsky (1992)) It offers a simple and intuitively appealing approach to pricing options. Although the computations can be tedious if many periods are considered, only algebra is needed to derive the pricing formulas of *BOPM*. Most important, it is preference free. This means the risk preferences of investors are not relevant in pricing the option. (Cox and Rubinstein (1985), Bookstaber (1987), Dubofsky (1992))

Under certain conditions, the solutions of *BOPM* approximate the solution of more complex models like the Black-Scholes Model (B-S). Simplification in *BOPM* is achieved by considering time in discrete periods, in contrast with continuous time in the B-S model. In addition *BOPM* assumes that the price change follows a binomial process, i.e., the price can go up by a certain factor u or go down by a certain factor d per period.[2] This contrasts with the diffusion process assumed in B-S. This simply means the price does not jump from one level to another but changes continuously, taking on every value in between. (Bookstaber (1987)) This feature of *BOPM* makes it particularly appropriate for the pre-termination option since the depositor has only two choices–either he pre-terminates or he does not. Furthermore, since the pre-termination can be exercised anytime, it should be priced as an American option. *BOPM* works for American and European options, but B-S applies only to the latter, i.e., options which can only be exercised at expiration.

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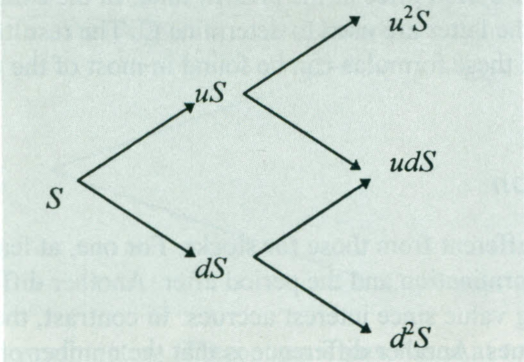
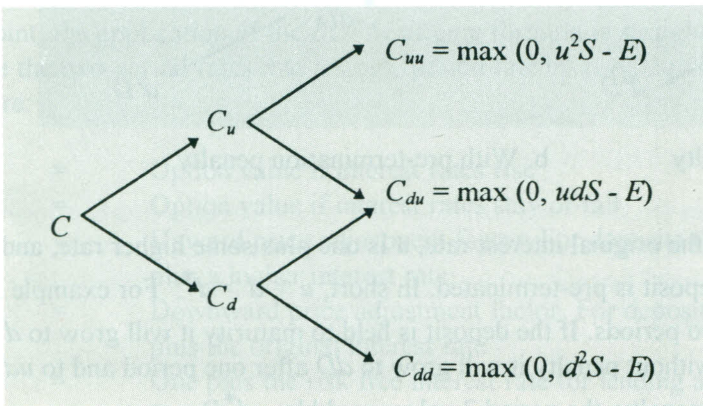
[2] Dubofsky says it is not necessary that u and d be constant through all periods. They only need to be known each period. A constant u and d simplifies the expression of the general formula.

The binomial process for a stock in a two period case are shown below, where S is the starting price.



After one period the price could go up to uS or go down to dS. Having gone up in period one, it could rise again in period two to u^2S or go down to udS. Note that udS is equal to duS. The outcomes are analogous to counting the number of heads or tails in several tosses of a coin. Cox and Rubinstein term it a stochastic process that follows a stationary multiplicative random walk.

The corresponding pattern of call option prices on this stock is illustrated below. The C's are the option prices at various point in time for a given sequence of price changes and E is the exercise or strike price.



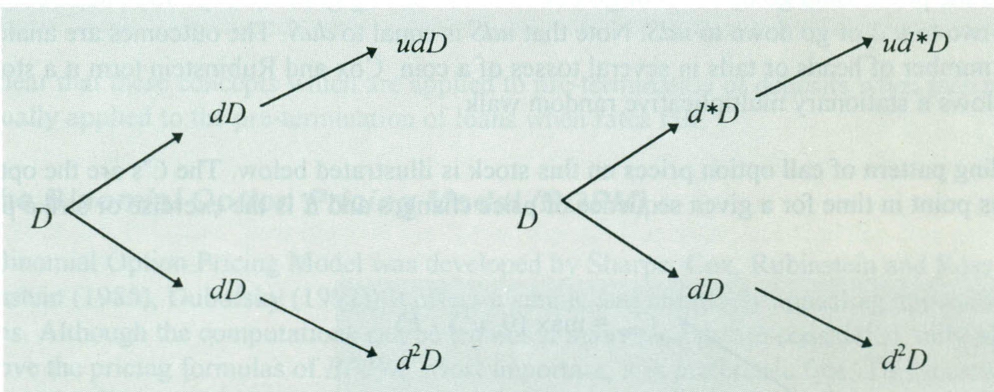
The expressions on the right side simply state that the price of the option is the higher of zero or the excess of the market price over the exercise price. For example if the exercise price is P10 per unit and the market price is P12 per unit, then a call option on one unit is worth P2. On the other hand if the market price were P9 per unit, the option would be worthless. Option price cannot be negative.

The usual approach to pricing options is to find a portfolio of securities whose pay-off profile exactly matches the pay-off profile of the option. This is true of the *BOPM*, as well as the Black-Scholes model. If the two have exactly the same pay-off profile, they should have the same price if arbitrage is to be prevented. Arbitrage is defined as an opportunity for riskless profit without any investment. The portfolio of securities could take the form stocks plus borrowing, or lending and borrowing. An equivalent approach is to form a portfolio of options and other securities and determine a price for the option that precludes arbitrage. In both approaches, the unknown option price is determined from the known prices of stocks, bonds, etc. The common simplifying assumptions are usually made, e.g., no taxes and transactions cost, short selling is permitted, etc. (Cox and Rubinstein (1985), Dubofsky (1992), Bookstaber (1987))

Determining the option price C using *BOPM* is a recursive process where the option prices from the most distant period are worked back to determine the option price at the present time. In the example, C_{uu}, C_{ud} and C_{dd} are used to determine C_u and C_d, and the latter are used to determine C. The resulting formulas are presented in the next section. The derivation of these formulas can be found in most of the references listed at the end of this paper.

3. Pricing the Pre-Termination Option

The pay-off profile from a deposit is slightly different from those for stocks. For one, at least two periods must be considered, the period before the pre-termination and the period after. Another difference is that later values are always greater than the starting value since interest accrues. In contrast, the price of stocks could be higher or lower than their starting values. Another difference is that the number of branches is less than the branches in a pay-off profile for stocks for the same number of periods. This is shown below.

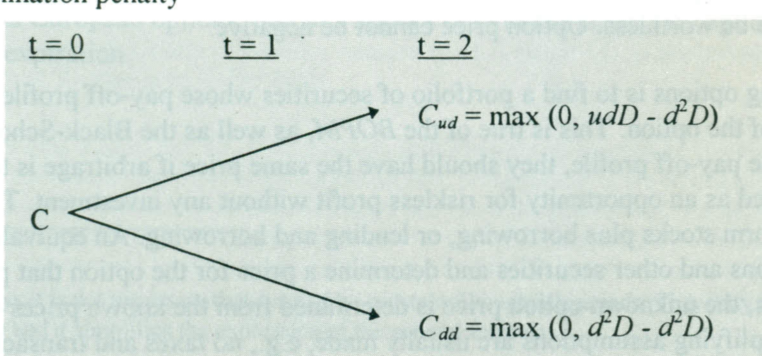


a. No pre-termination penalty   b. With pre-termination penalty

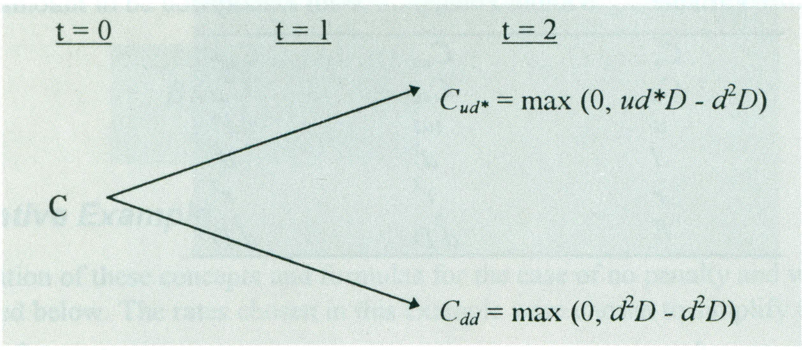
D is the initial deposit, d is one plus the original interest rate, u is one plus some higher rate, and d^* is one plus the penalty interest rate if the deposit is pre-terminated. In short, $u > d > d^*$. For example a deposit is made with an original maturity of two periods. If the deposit is held to maturity it will grow to d^2D after two periods. If it is pre- terminated without penalty it will grow to dD after one period and to udD after two periods. If there is a pre-termination penalty, the period 2 value would be ud^*D.

The corresponding pattern of call option prices is shown below.

a. No pre-termination penalty



b. With pre-termination penalty



Several points are worth noting. The option is purchased at $t = 0$ at price C. It is exercised at $t = 1$ but the pay-off takes place at $t = 2$. This differs from stock options in a significant way. In stock options, the pay-off from an exercise is immediate. One can exercise the option and immediately turn around and sell the stock at a profit. In the case of deposits, the pay-off comes some time after the exercise. However, what happens in $t = 2$ is completely determined by what happens in $t = 1$. This means that in analyzing the call option, the two periods can be treated as one, and only two branches are formed.

The other point to note is that the exercise price in the pre- termination option is equal to d^2D. This is because if the deposit is kept to maturity d^2D will be received. The option will be exercised only if the pay-off is greater than d^2D. This implies that C_{dd} is always zero.

From this point, the application of the *BOPM* pricing formula is straight-forward. All that needs to be done is to collapse the two-period rates into a single period rate by compounding them. The variables required for *BOPM* are:

C_u	=	Option value if interest rates rise
C_d	=	Option value if interest rates stay or fall
u	=	Upward price adjustment factor. For deposits this corresponds to one plus a higher interest rate.
d	=	Downward price adjustment factor. For deposits this corresponds to one plus the original interest rate.
r	=	One plus the risk free interest rate for lending and borrowing.
E	=	Exercise price.
$d*$	=	One plus the penalty interest rate, $d > d^*$.

$u > r > d$ is required to preclude arbitrage.

The counterparts of these variables in the case of the pre-termination option are shown below.

Variable	Without Penalty	With Penalty
C_u	C_{ud}	C_{ud*}
C_d	C_{dd}	C_{dd}
u	ud	$ud*$
d	d^2	d^2
r	r^2	r^2
E	d^2D	d^2D

The value for C_{ud} is max(0, udD - d^2D) for pre-termination without penalty, max(0, $ud*D$ - d^2D) for pre-termination with penalty, and C_{dd} is zero in both cases.

The *BOPM* formulas are:

a. Call price at t = 0

$$C = \frac{pC_u\,(1-p)C_d}{1+r} \qquad \text{where} \qquad p = \frac{r-d}{u-d}$$

b. *h*, the hedge ratio

$$h = \frac{C_u - C_d}{(u-d)D}$$

The hedge ratio is always one because the exercise price is set at d^2D. This means that the underlying security value and the option price always move one-for-one with each other. Intuitively one must sell (or buy) as much options as there is "excess" interest received (or paid) to establish a riskless hedge. The formal proof of this is presented below.[3]

[3] A riskless hedge portfolio composed of the underlying asset and the option is formed. In this case the depositor holds the deposit and sells the call option. The hedge ratio must be chosen such that the outcome is the same whatever the state of nature is, i.e., whether rates rise or stay/fall. Thus, one can write: $hudD - C_u = hd^2D - C_d$. Using the definition of C_u and C_d this can be further expanded to: $hudD$ - max ($udD - d^2D$, 0) = hd^2D - max ($d^2D - d^2D$, 0). Simplifying and solving for h, one gets:

$$h = \frac{udD - d^2D}{udD - d^2D} = 1.$$

c. B, amount to be borrowed if the resulting amount is negative, or invested if the amount is positive.

$$B = \frac{uC_d - dC_u}{(u-d)r}$$

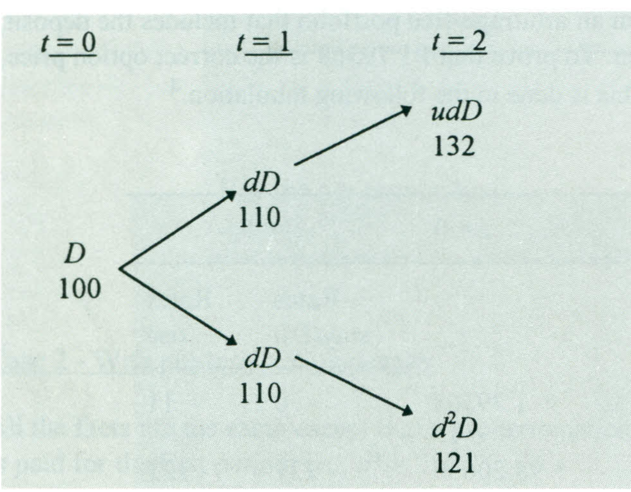
4. Illustrative Example

The application of these concepts and formulas for the case of no penalty and with penalty are demonstrated below. The rates chosen in this example were chosen to simplify the exposition.

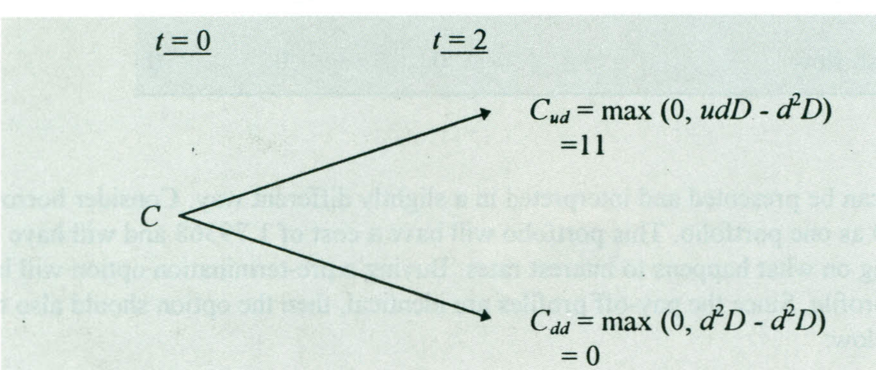
<u>Case 1 - No pre-termination penalty</u>

D	=	P100	initial deposit
d	=	1.10	or an interest rate of 10% per period
u	=	1.20	or an interest rate of 20% per period
r	=	1.11	implying a risk free borrowing/lending rate of 11% per period.

The pay-off profile of the deposit is



The pay-off profile of the option is:



The objective is to determine C, the option price at $t = 0$. The following formulas will give the answer. Note that the counterpart values will have to be used.

$$C = \frac{pC_u + (1-p)C_d}{1+r} = \frac{(0.2009)11 + (0.7991)0}{1.2321} = 1.79368$$

$$p = \frac{r-d}{u-d} = \frac{1.2321 - 1.21}{1.32 - 1.21} = 0.200909$$

$$1 - p = 1 - 0.200909 = .79909$$

$$C_u = \max(0, 132 - 121) = 11$$

$$C_d = \max(0, 121 - 121) = 0$$

$$B = \frac{uC_d - dC_u}{(u-d)r} = \frac{(1.32)0 - (1.21)11}{(1.32 - 1.21)1.2321} = -98.2063$$

This means P98.2063 must be borrowed to form an arbitrage-free portfolio that includes the deposit, borrowing and selling the pre-termination option. To prove that P1.79368 is the correct option price, it must be shown that arbitrage is not possible. This is done in the following tabulation.[4]

	$t = 0$	$t = 2$	
		Rates stay/fall	Rates rise
Sell pre-termination option	+ 1.79368	0	- 11
Borrow at 11%	+ 98.20630	- 121	- 121
Invest at 10%/20%	- 100.00000	+ 121	+ 132
Net cash flow	0	0	0

The same set of numbers can be presented and interpreted in a slightly different way. Consider borrowing 98.2063 and investing 100 as one portfolio. This portfolio will have a cost of 1.79368 and will have a pay-off either 0 or 11 depending on what happens to interest rates. Buying a pre-termination option will have exactly the same pay-off profile. Since the pay-off profiles are identical, then the option should also cost 1.79368. This is shown below:

[4] Some slight rounding errors remain.

Borrowing/investing portfolio

	t = 0	t = 2	
		Rates stay/fall	Rates rise
Borrow at 11%	+ 98.20630	- 121	- 121
Invest at 10%/20%	- 100.00000	+ 121	+ 132
Net cash flow	-1.793680	0	+ 11

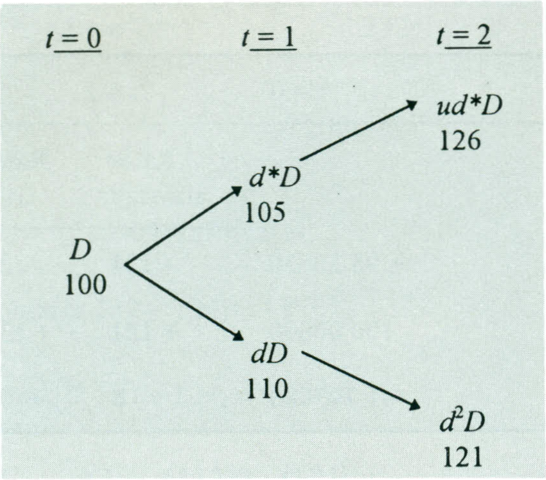
Buying the pre-termination option

	t = 0	t = 2	
		Rates stay/fall	Rates rise
Buy the pre-termination option	- 1.79368	0	+ 11

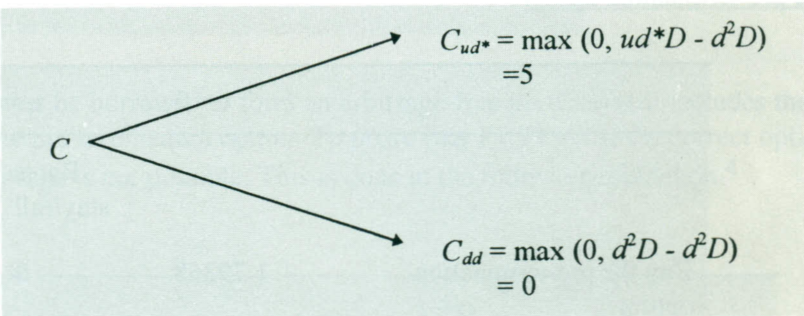
Case 2 - With pre-termination penalty

All the facts are the same except that a pre-termination penalty is imposed. Only one-half the original rate is paid for the first period, i.e., $d^* = 5\%$.

The pay-off profile of the deposit is



The pay-off profile of the option is:



The results are:

C	=	1.79368
p	=	.442
$1\text{-}p$	=	.558
C_u	=	5
C_d	=	0
B	=	-98.2063

The following table shows that 1.79368 is the correct price since arbitrage is not possible.

	t = 0	t = 2	
		Rates stay/fall	Rates rise
Sell pre-termination option	+ 1.79368	0	- 5
Borrow at 11%	+ 98.20630	- 121	- 121
Invest at 10%/20%	- 100.00000	+ 121	+ 126
Net cash flow	0	0	0

The unexpected, counter-intuitive result is the option prices are equal in the two cases. It seems logical to expect that the option price should be lower where a penalty is imposed on its exercise. Mathematically, this result is easy to trace. It is the result of assuming the exercise price to be equal to d^2D. As pointed out earlier this results in a C_d always equal to zero and a hedge ratio, h, always equal to 1. If so, u does not figure in the price of the option. The proof follows.

If $C_d = 0$, then

$$C = \frac{pC_u + (1-p)C_d}{r} \quad \text{simplifies to} \quad C = \frac{pC_u}{r} \tag{1}$$

Also, if $C_d = 0$, then $h = \dfrac{C_u}{uD - dD}$ and

$$C_u = h(uD - dD) = h(u-d)D \tag{2}$$

Substituting (2) for C_u in (1) gives

$$C = \frac{pC_u}{r} = \frac{\left(\dfrac{r-d}{u-d}\right)h(u-d)D}{r} \tag{3}$$

$$= \frac{(r-d)hD}{r}$$

Equation (3) gives exactly the same result as the original *BOPM* formula. It may appear that the term u does not appear in the formula but u is implicit in h. However in the case of the pre-termination option h is always equal to 1, which means it can be dropped from the formula. This appears to explain why the upper half of the pay-off profile doesn't figure at all in the price of the option.

Note that this result is not unique to the pre-termination option. A stock option priced under *BOPM* would have the same option price even if the pay-off profiles are different provided $h = 1$. Consider the two cases as the pay-off profiles of two stock options both of which have an exercise price of 121; this gives identical option prices for two stock options with different pay-off profiles. There is nothing implausible about this particular configuration of numbers. However while it is possible for the stock option price to behave this way, it is an exception since h is generally not equal to 1. For the pre-termination option, it appears to be the rule since h is always equal to 1. Whether this result makes sense is an open question. At this point three possibilities are suggested. These are discussed in the concluding remarks.

5. Concluding Remarks

There are three possible reasons to explain the unexpected equality of option prices. These are:

a. The adaptation is wrong. Setting the exercise price at d^2D may not be correct although there appears to be no logical alternative. d^2D is the minimum that the depositor will receive if he does not pre-terminate; any exercise of the option should yield more than d^2D. The case for d^2D appears strong. Moreover this argument doesn't address the case where stock options having different pay-off profiles have the same price, as the basic *BOPM* was used in pricing these stock options. Nevertheless, if there is an error it is likely to lie in setting the correct exercise price.

b. *BOPM* "breaks down" in "special" cases where $C_d = 0$ and $h = 1$ though the references have not mentioned any restriction that limits the validity of *BOPM*. In fact, a number of them (Bookstaber (1987), Brealey and Myers (1988)) state that *BOPM* is a flexible model that can be applied in cases where more complex models can't be used.

c. The equal prices are correct even if they run counter to intuition. There may be a valid argument to explain why they should be equal. The assumption of risk neutrality commonly utilized in option pricing was considered but it doesn't seem to be relevant.

This paper winds up on a less than entirely satisfying note. While a price for the pre-termination option has been determined and it has been shown that these are the "correct" prices in the sense that arbitrage is precluded, there is still no satisfactory explanation why the option price is invariant relative to pre-termination penalties. This is a matter that may merit further analysis.

References

1. Bookstaber, Richard M, (1987) *Option Pricing and Investment Strategies.* Reading MA: Addison-Wesley.

2. Brealey, Richard A. and Stewart C. Myers, (1988) *Principles of Corporate Finance.* New York: McGraw Hill, Chs. 20-21.

3. Cox, John C. and Mark Rubinstein, (1992) *Option Markets.* Englewood Cliffs, NJ: Prentice-Hall, Chs. 1-5.

4. Dubofsky, David A., (1992) *Options and Financial Futures - Valuation and Uses.* New York: McGraw Hill, Chs . 1-6.

5. Gibson, Rajna, (1991) *Option Valuation - Analyzing and Pricing Standardized Option Contracts.* New York: McGraw-Hill, Ch. 3.

6. Hull, John, (1991) *Introduction to Futures and Options Markets.* Englewood Cliffs, NJ, Ch. 13.